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PARETO-OPTIMAL FORMS OF AXISYMMETRIC BODIES MOVING AT HIGH SUPERSONIC SPEEDS*

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The problem of optimizing the shape of an axisymmetric body with respect to three criteria, namely the wave resistance and radiative and convective heating during its motion in the atmosphere at supersonic speeds, is considered. The optimal solution of the problem with many criteria is described as the Pareto-optimal solution. A system of integrodifferential equations is obtained for determining the optimal solution. A numerical algorithm of the solution is proposed, and used as the basis for the numerical determination of the optimal body profiles ensuring that one of the following four criteria is satisfied: 1) minimum radiative heating of the body; 2) minimum sum of radiative and convective heating; 3) minimum value of the total heating and wave resistance; 4) minimum value of convective heating and wave resistance. Pareto diagrams are given for the solutions obtained, and a comparison is made of the total heating of the optimal and reference bodies, namely of a sphere and a paraboloid.

When choosing the shape of aircraft, one must take into account a large number of physical factors (the wave resistance, surface heating, etc.) characterizing the condition of the flight. The problem of choosing a shape that is optimal with respect to one of the criteria was studied in detail in e.g. /1-5/. It was found, in particular, that the shape of the body with minimum wave resistance is quite different from the shape of a body with minimum heating. The question arises of which body should be regarded as optimal when several criteria have to be taken into account, for example the wave resistance and the heating. We propose below that the concept of Pareto optimality /6/ should be adopted in the case of optimization with respect to several criteria.

We consider three functionals: the radiative heating $Q_R = \Phi_1$, the wave resistance $R = \Phi_2$ and convective heating $Q_C = \Phi_3$. The distribution of radiative heat flux over the surface of a non-disintegrating axisymmetric body is sought by solving the equations of gas dynamics for a non-viscous, bulk-radiating shock layer around the body, based on the method of a strongly compressed layer. The functional Φ_1 for radiative heating in given in /5/. A hypersonic theory of Newton /1/ is used to determine the wave resistance. Convective heat exchange is found using /2, 3/ the hypothesis of local similarity in the flow past a plate and the hypersonic Newton's theory, and the functional Φ_3 was obtained for bodies with flat nose and limited inclination of the side surface. As a result, we obtain the following expressions for the functionals:

$$\Phi_i = \int_0^1 L_i(u, u^{\cdot}, t)\,dt + f_i(u_0), \quad i = 1, 2, 3 \tag{1}$$

$$L_1 = L_2(u^{\cdot})(1 - W(u^{\cdot}, t)), \quad L_2 = \frac{\varepsilon^3 u^{\cdot 3}}{1 + \varepsilon^2 u^{\cdot 2}}$$

$$L_3 = u^{\varkappa} \Pi^a \sqrt{(1 - \Pi^b)(1 + \varepsilon^2 u^{\cdot 2})}$$

$$a = \frac{1}{\gamma} + \omega\eta b, \quad b = \frac{\gamma - 1}{\gamma}, \quad \Pi = \frac{\varepsilon^2 u^{\cdot 2}}{1 + \varepsilon^2 u^{\cdot 2}}, \quad u^{\cdot} = \frac{du}{dt}$$

$$W = P^{-1/\alpha}, \quad P = 1 + B\frac{\Pi^\alpha}{\sqrt{1 - \Pi}} \int_t^1 \frac{dt}{\sqrt{1 - \Pi}}, \quad \alpha = n + 4$$

$$f_1 = f_2 = \frac{1}{2}\varepsilon u_0^2, \quad f_3 = \varepsilon \frac{\delta_*}{\varkappa + 2} u_0^{\varkappa + 1}$$

$$\delta_* = \frac{1}{a}\left(\frac{\gamma - 1}{\gamma + 1}\right)^{1/2} \left(\frac{2}{\gamma + 1}\right)^{\omega(1-\gamma)+1}$$

$$u = \frac{y}{r}, \quad t = \frac{x}{l}, \quad \varepsilon = \frac{r}{l}, \quad B = 2(n + 4)\Gamma$$

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Here the assumption that the velocity of the gas at the edge of the flat nose is the same as the speed of sound, results in restricting the maximum value of the pressure on the side surface /3/, and this corresponds to restricting the maximum angle of inclination of the side surface:

$$\varepsilon u^{\cdot} \leqslant w, \quad w^2 = \frac{\Pi_*}{1-\Pi_*}, \quad \Pi_* = \left(\frac{2}{\gamma+1}\right)^{1/b}$$

Here and in (1) x, y are the coordinates along the axis of the body and along its normal, l is the length of the body, r is the radius of the middle section, ε is the relative thickness of the body, $u_0 = u(0)$ is the radius of the flat nose section, γ is the ratio of the specific heats, ω is the power index in the relation connecting the viscosity and temperature, and the constants $\varkappa$ and η are determined by the type of flow in the boundary layer. For laminar flow $\varkappa = 2$, $\eta = 1$, and for turbulent flow $\varkappa = 5/4$, $\eta = 1/4$, n is the power index in the relation connecting the Planck absorption coefficient and the temperature, Γ is the radiation parameter /4, 5/, equal to the ratio of the characteristic radiative flux q_* behind the discontinuity to the kinetic energy flux $(0.5\rho_\infty V_\infty^3)$ of incoming gas flow, and ρ_∞, V_∞ are the density and velocity of the incoming gas.

Let us recall the concept of Pareto-optimality /6/. Suppose there are two criteria, Φ_1 and Φ_2, which have to be minimized. The solution will be Pareto-optimal if one of the criteria cannot be improved without adversely affecting the other criterion. If the problem has an infinite number of such solutions, then the line formed by them in the plane $\Phi_1\Phi_2$ is a Pareto diagram. The question arises of how to obtain this set of solutions. The determination of two extreme points of the line is obvious. The points represent the solutions minimizing the functional Φ_1 only, and the functional Φ_2 only. Moreover, the Pareto-optimal solution can be interpreted as the minimum of the functional Φ_1 for a prescribed value of Φ_2. The same solution can also be interpreted as a minimum of the functional Φ_2 for a prescribed value of Φ_1. The analogy between the two-criteria problem and the problem with an isoperimetric constraint is clear. Thus, by minimizing the linear combination of the functionals we obtain a set of solutions which are Pareto-optimal. All the above can be generalized to the case of several criteria.

We shall formulate the three-criteria problem of the optimal shape of a body in a supersonic gas flow as follows. It is required to find, amongst the functions $u(t)$ satisfying the differential condition $\varepsilon u^{\cdot} \leqslant u$ and the boundary condition $u(t=1) = 1$, the functions which minimize the convolution of the functionals $\Phi = \rho_1\Phi_1 + \rho_2\Phi_2 + \rho_3\Phi_3$. The solution is determined here by the value of the weighting factors $\rho = \{\rho_1, \rho_2, \rho_3\}$. Usually the most important coefficient is assumed to be equal to unity, and the values of the remaining parameters are chosen from the interval $(0, 1)$, and the higher the priority of the functional Φ_i, the greater the value assigned to the corresponding coefficient ρ_i. The final value of the vector ρ is found when solving the problem, using the Pareto diagram.

Using the method of Lagrange multipliers /1, 7/ we can reduce the problem to that of finding the unconditional extremum of the functional

$$\psi = \min\left\{\int_0^1 K(u, u^{\cdot}, t)\,dt + f(u_0)\right\}, \quad u(1) = 1 \tag{2}$$

$$K = L + \lambda(t)\left[\varepsilon u^{\cdot} - w + \alpha^2(t)\right], \quad L = \sum_{i=1}^{3} \rho_i L_i, \quad f = \sum_{i=1}^{3} \rho_i f_i$$

The Euler equations of problem (2) have a first integral. The first integral and three Euler equations for the unknown functions $u(t)$, $v(t)$ $\lambda(t)$ and $\alpha(t)$ can be written in the form $(H,\ l = \text{const})$:

$$F(u, v, y, G) + \lambda(w - \alpha^2) = H, \quad K_\lambda^{\cdot} = v - w + \alpha^2 = 0 \tag{3}$$

$$y = \int_t^1 \sqrt{1+v^2}\,dt, \quad l = y(0), \quad K_\alpha^{\cdot} = 2\lambda\alpha = 0$$

$$G = \int_0^t \frac{B}{\alpha} u v^2 \left(\frac{v^2}{1+v^2}\right)^{\alpha+1/2} P^{-1-1/\alpha}\,dt, \quad \varepsilon u^{\cdot} = v$$

$$F = \frac{1}{1+v^2}\left\{2\rho_1 L_1 + 2\rho_2 L_2 + \rho_3 L_3\left[2a + b - 1 - b\left(1 - \left(\frac{v^2}{1+v^2}\right)^b\right)^{-1}\right]\right\} +$$

$$\rho_1\left\{u\,\frac{B}{\alpha}\,y\left(\frac{v^2}{1+v^2}\right)^{\alpha+3/2} P^{1-1/\alpha}(2\alpha + v^2) - G(1+v^2)^{-1/2}\right\}$$

We see from system (3) that the extremum for $t > 0$ can consist of the arcs of the following type:

1°. A segment of a two-sided extremum

$$\lambda = 0,\ F(u,\ v,\ y,\ G) - H = 0,\ \alpha^2 = w - v \tag{4}$$

and condition $K_{vv} \geqslant 0$ must hold on this solution.

2°. A segment of the boundary extremum

$$\alpha = 0,\ v = w,\ \lambda = (H - F)/w \tag{5}$$

and condition $\lambda \geqslant 0$ must hold on solution (5).

We join these arcs in the following manner.

1) at the points of intersection of the lines $F - H = 0$ and $v - w = 0$ in the (u, v) plane, in which case the Weierstrass-Erdman conditions /7/ hold automatically;

2) at the angle points using the Weierstrass-Erdman condition /7/

$$[(u^{\cdot}K_{u^{\cdot}} - K)]\,\delta t + [K_{u^{\cdot}}]\,\delta u = 0$$

(the square brackets denote the difference between the value calculated to the left and right of the angle point). Using Eqs.(4) and (5), we can write the condition in the form

$$(vL_v - L - wL_v)\,|_{(u,\,v)} + L\,|_{(u,\,w)} = 0$$

The condition $u(1) = 1$ and the condition of transversality

$$(T(u,\ v,\ l) + \lambda(t))\,|_{t=0} = 0 \tag{6}$$

$$T(u,v,l) = \Big\{\rho_1 u\,\Big\{\frac{v^4+3v^2}{(1+v^2)^2}(1-W) + \frac{Bl}{\alpha}\Big(\frac{v^2}{1+v^2}\Big)^{\alpha+1} P^{-1-1/\alpha}(2\alpha+v^2) - 1\Big\} +$$
$$\rho_2 u\,\frac{v^2-1}{(1+v^2)^2} + \rho_3\Big\{L_3\,\frac{1}{v(1+v^2)}\Big(2a + b + v^2 -$$
$$b\Big(1 - \Big(\frac{v^2}{1+v^2}\Big)^b\Big)^{-1}\Big) - \delta_*\,\frac{\varkappa+1}{\varkappa+2}\,u^{\varkappa}\Big\}\Big\}$$

must hold at the ends of the extremal curve.

Using Eqs.(4) and (5), we shall write condition (6) for the arcs of different type thus:

$$1°.\ T(u,v,l)\,|_{t=0} = 0;\quad 2°.\ \Big(T(u,w,l) + \frac{H-F}{w}\Big)\Big|_{t=0} = 0$$

Hence we have reduced the problem of solving problem (2) to that of solving the following system of integrodifferential equations with unknown functions $u(t)$, $v(t)$ and unknown H, l:

$$F_1(u,\ v,\ y(u,\ v),\ G(u,\ v)) = 0 \tag{7}$$

$$\varepsilon u^{\cdot} = v,\quad l = \int_0^1 \sqrt{1+v^2}\,dt,\quad u(1) = 1,\quad T_1(u(0),\ \ v(0), l) = 0 \tag{8}$$

Here $F_1 = F - H$, $T_1 = T$ for arcs of type 1°, and $F_1 = v - w$, $T_1 = T + (H - F)/w$ for arcs of type 2°.

The functions $\alpha(t)$, $\lambda(t)$ are expressed explicitly in terms of $u(t)$, $v(t)$.

We construct the solution of system (7) and (8) as follows. Instead of the variable t we use u, which is possible by virtue of the monotonic form of the relation $u(t)$. We introduce the functional

$$Q(H, l, v(u)) = \Big(\varepsilon\int_{u_0}^1 \frac{du}{v} - 1\Big)^2 + \Big(\varepsilon\int_{u_0}^1 \frac{\sqrt{1+v^2}}{v}\,du - l\Big)^2 \geqslant 0$$

The functional Q reaches a minimum, equal to zero, on solutions of the system of integro-differential Eqs.(7), (8), by virtue of Eq.(8). It is clear that the pair of numbers H, l and the function $v(u)$, which reduce the functional Q to zero and satisfy the system of equations $F_1 = 0$, $T_1 = 0$, are also a solution of system (7), (8). Therefore, instead of solving the system of integrodifferential equations, we propose to seek a solution $v(H, l, u)$ of integral Eq.(7) which will satisfy the boundary condition $T_1 = 0$, and to find the values of parameters H, l, for which $Q(H, l, v(H, l, u)) = 0$, after which all functions of system (7), (8) can be found with help of a single quadrature.

We shall seek the solution of this problem using the method of successive approximations: by directional inspection we construct a sequence of points (H^k, l^k) in parameter plane H, l, and a corresponding sequence of functions $v^k(H^k, l^k, u)$ which ensure, in the limit, a minimum of the functional Q. The solution of integral Eq.(7) is reduced, for every k, to solving a system of N algebraic equations with a single variable in each equation.

Using the above algorithm we determined numerically the profiles of axisymmetric bodies ensuring that one of the following criteria holds:

1) a minimum of radiative heating $Q_R = \Phi_1$.

2) the smallest sum of the radiative and convective heating

$$Q = Q_R + Q_C, \ Q_C = \rho_3\Phi_3$$

3) a Pareto-minimum of the convective heating $Q = Q_R + Q_C$ and the wave resistance $R = \Phi_2$ $(\Phi = Q + \rho_2 R)$.

4) a Pareto minimum of the convective heating Q_C and the wave resistance R.

All computations were carried out using the following values of the parameters $n = 0$ $(\alpha = 4)$; $\gamma = 1{,}1$; $\eta = 1$, $\varkappa = 2$ (laminar boundary layer).

Problem 1 was solved earlier /4/ numerically, using the local variation method. In the case of smooth solutions, complete agreement of the solutions exists (to within the thickness of the lines in the figure), and this implies the suitability of the algorithm developed above.

The algorithm was realized in Fortran-4 (the machine-independent version). The time taken to compute a single version depends on the initial conditions, on the parameters of the problem, and on the number N of partition points. Computing a single version on an ES-1040 digital computer with $N = 50$ does not take more than 3 minutes. The algorithm does not take more time than the method of local variations and is, in addition, numerically analytic, and this is the reason why it is preferable to other numerical methods (e.g. to the method of local variations). The algorithm makes it possible to obtain solutions with constraints imposed on the derivative, and with a discontinuity in the derivative. This made is possible to determine the shape of the body that is optimal with respect to the sum of radiative and convective heating. The flexibility of the numerically analytic method enabled us to obtain a solution for the two-criteria problem (convective heating and wave resistance) of optimizing an axisymmetric body where the optimal generatrix contains a segment of the boundary extremum. In the last problem the proposed algorithm is relatively fast. Its speed, as compared with the general case of the algorithm, is ensured by the fact that the optimization is carried out over a single parameter H and not over H and l as in the general case. For $N = 100$ a single version is computed in about 2 minutes on a CM-4 computer, and in a few seconds on an ES-1040 computer.

After solving the variational problem, we calculated the dimenionless heat fluxes Q_R, Q_C and Q, integral over the surface of the body defined above (the dimensional fluxes are referred to the quantity $\frac{1}{2}\rho_\infty V_\infty^3 S$) where S is the area of the middle cross-section of the body.

We also compared the total heating of the optimal bodies Q_{opt} with that of the reference bodies (a sphere and a paraboloid) Q_{eq}, using the formula

$$q = (|\, Q_{\text{eq}} - Q_{\text{opt}}\, |/Q_{\text{opt}})\cdot 100\%$$

The results of computing the shape of the bodies with least total heating $Q = Q_R + Q_C$, $Q_C = \rho_3\Phi_3$ are given in Fig.1 (problem 2). The computations were carried out for $\varepsilon = 1$ with $B = 1$ (curves *1-3*) and $B = 10$ (curves *4-6*).

The convective and radiative heating of a hemisphere was computed earlier /8/ for various flow modes. It was shown that the ratio Q_C/Q_R is contained within the limits from 0.03 to 2. The distribution of radiative and convective heat fluxes at the stagnation point in given in /9/ along the trajectories for bodies of different forms (spherical, a sphere-cylinder and a sphere-cone). In all cases the predominance of radiative fluxes $(Q_C/Q_R = 0{,}25)$ was observed, although the fluxes themselves can vary over an arbitrary wide range.

Using the results of /8, 9/, we chose the parameters ρ_3 determining the ratio of the radiative and convective fluxes towards the body in such a manner, that the relations $Q_C/Q_R = 1$ (curves *2* and *5* of Fig.1) and $Q_C/Q_R = 2$ (curves *3* and *6* of Fig.1) hold for a sphere. Curves *1* and *4* correspond to the problem of minimum radiative heating $(\rho_3 = 0)$ with a constraint imposed on the inclination of the generatrix. The extremal curves shown in Fig.1 have an end face in all cases at $t = 0$ (the radius of the face decreases as the irradiation parameter B decreases) and consists of arcs of type 1° and 2° (curves *4*, *5*). We note that when $B \leqslant 1$, then the size of the face at $t = 0$ is not large and is indistinguishable within the scale Fig.1. Near $t = 0$ and $t = 1$ the generatrix becomes a segment of the straight line $v = w$ (type 2°), and near $t = 1$ the length of the straight line section increases as B decreases and as ρ_3 $(Q_C/Q_R \in [0, 2])$ increases.

The table give, for $\varepsilon = 1$, the values of the convective heating Q_C and radiative heating Q_R for bodies of a shape that is optimal with respect to the total heating, and for the reference bodies (a sphere or a paraboloid) for various values of ρ_3. The table also gives the loss, in percents of the total heating on a sphere q_1 and paraboloid q_2 as compared with the optimum body.

Fig.1 shows that the form of the body that is optimal with respect to the total heating changes very little as $\rho_3\ (Q_C/Q_R \in [0, 2])$ increases, and is close to the form of a body that is optimal with respect to radiative heating only. The table shows that the optimization is governed, basically, by the radiative heating, while convective heating either decreases a little, or even increases. This can be explained by the fact that the functional of radiative heating Q_R changes appreciably even when the form of the body undergoes a small change, while the opposite is true for the functional Q_C.

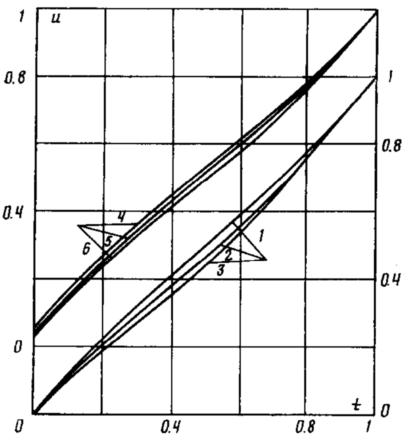

Fig.1

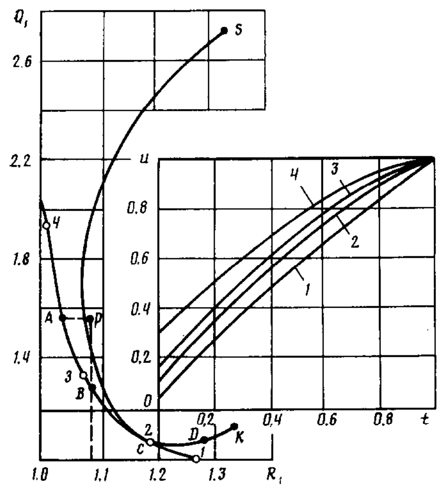

Fig.2

Fig.2 shows the extremal curves $u(t)$ of problem 2 (the solution is Pareto-optimal) for $\varepsilon = 1, B = 10, \rho_3 = 0.385$. Such a value of ρ_3 corresponds to the ratio of convective and radiative fluxes towards the sphere $Q_C/Q_R = 0.25$. Curves *1-4* in Fig.2 correspond to $\rho_2 = 0; 0.4; 1; 6$. Fig.2 also shows the Pareto diagram corresponding to these bodies, i.e. the curve *ABCD*. The normalized values of the wave resistance $R_1 = \Phi_2(\rho_2)/\Phi_2(\rho_2 = \infty)$ are plotted along the abscissa, and the normalized total heating $Q_1 = Q(\rho_2)/Q(\rho_2 = 0)$ along the ordinate. The points *1-4* in this diagram correspond to the optimal bodies $u(t)$ (1—4). The corresponding values for spherically ended blunt cones with various radii of bluntness (the line *KS*, *K* is a cone and *S* is a sphere), and for the paraboloid (point *P*) are also given here.

We see from Fig.2 that any shape, Pareto-optimal with respect to a sphere, provides a gain with respect to heating, as well as to the wave resistance. A paraboloid compared with the Pareto-optimal body is inferior, for the same wave resistance, by $\sim 28\%$ with respect to heating, and for the same heating $\sim 4\%$ with respect to the wave resistance. A segment *AB* exists, in which the optimal forms will be superior, as compared with the paraboloid, in both criteria. However, the segment *BD* is also of interest. For example, a body corresponding to the point *C* in the Pareto diagram will lose by $\sim 10\%$ in wave resistance, and gain $\sim 50\%$ in heating, as compared with the paraboloid.

Fig.3 shows the results of a numerical solution of problem 4. The Pareto-optimal forms $u(t)$ are shown for $\varepsilon = 1$ and for six values of the parameter ρ_2. The form 1 $(\rho_2 = 0)$ is optimal only with respect to convective heating, and form 6 $(\rho_2 = \infty)$ only with respect to the wave resistance. The forms $2—5$ $(\rho_2 = 0.05; 0.07; 0.1; 0.3$ respectively (are Pareto-optimal with respect to wave resistance and convective heating. The curves are composed of arcs of type 1° (curves *1, 2* and *6*) and type 2°-1° (curves *3, 4* and *5*). Fig.3 shows their Pareto diagrams. The normalized value of the wave resistance $R_1 = \Phi_2(\rho_2)/\Phi_2(\rho_2 = \infty)$ is plotted along the abscissa, and that of convective heating $Q_1 = \Phi_3(\rho_2)/\Phi_3(\rho_2 = 0)$ along the ordinate. The points 1, 2, ..., 6 in Fig.3. The curve *AB* shows the values of R_1, Q_1 on a truncated cone, in which the radius of the end face of a truncated tip changes from 0.1 (point *A*) to 0.9 (point *B*).

The Pareto diagram enables one to estimate the gain or loss of the bodies, Pareto optimal for various ρ and reference bodies, and enables us to choose the most acceptable forms out of the set of the relatively optimal forms.

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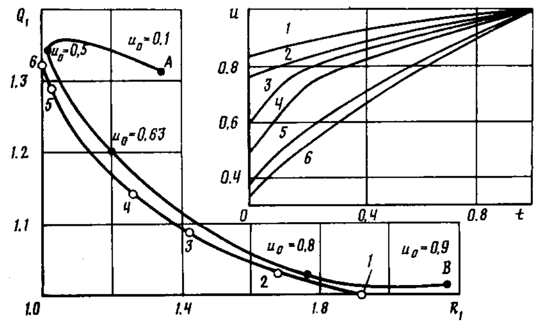

Fig.3

B	1			10		
ρ_s	0	0.632	1.264	0	1.542	3.084
			Sphere			
$Q_R\cdot10^4$	332	332	332	809	809	809
$Q_C\cdot10^4$	0	332	664	0	809	1618
$Q\cdot10^4$	332	664	996	809	1618	2427
			Paraboloid			
$Q_R\cdot10^4$	128	128	366	366	366	366
$Q_C\cdot10^4$	0	380	759	0	929	1859
$Q\cdot10^4$	128	508	887	366	1295	2225
	Body optimal with respect to total heating					
$Q_R\cdot10^4$	20.9	24.2	29.4	145	152	167
$Q_C\cdot10^4$	0	369	731	0	919	1819
$Q\cdot10^4$	20.9	393	760	145	1071	1986
q_1	510	29.1	16.8	458	51.0	22.2
q_2	149	68.6	30.8	152	20.9	12.0

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